A.P. Physics-C Name ________________________________

CALCULUS PRACTICE

This assignment is to be done in class without a calculator.

POWER RULE DERIVATIVES

For each of the following functions, find the derivative (dy/dx) with respect to x:

1. $y = 5x^2$ **2.** $y = 4x$ **3.** $y = 8x^3$

4. $y = 2$ **5.** $y = 2x^{-1}$ **6.** $y = 4x^3 + 2x - 1$

OTHER COMMON DERIVATIVES

For each of the following functions, find the derivative (dy/dx) with respect to x:

7. $y = \cos x$ **8.** $y = e^x$ **9.** $y = \ln x$

CHAIN RULE DERIVATIVES

Use the chain rule to help you find dy/dx for each of the following functions:

10. $y = (x^2 + 4)^3$ **11.** $y = e^{3x}$ **12.** $y = \sin(3x+1)$

PRODUCT RULE DERIVATIVES

Use the product rule to help you find dy/dx for each of the following functions:

13. $y = (x^2)(2x - 7)$ **14.** $y = 2x^3e^x$ **15.** $y = (5x^2 + 3x)(\ln\ x)$

MAXIMUM AND MINIMUM VALUES

Find the maximum y-coordinate reached by the following functions:

16. $y = -3x^2 + 12x$

17. $y = -2x^2 - 20x + 12$

INDEFINITE INTEGRALS

Use the idea of "un-doing a derivative" to find the following indefinite integrals:

18. $\int 3x^2 dx$

19. $\int (8x^3 + 2x)dx$

20. $\int (5x^3 - 1)dx$

DEFINITE INTEGRALS

Evaluate each of the following definite integrals:

21. $\int_0^2 (6x^2 + 3)dx$

22. $\int_1^5 (4x + 1)dx$

CALCULUS AND GRAPHS

Use calculus ideas to find the required graphical values of the following functions:

23. Find the instantaneous rate of change for the function $y = x^2 + 3x - 5$ at the point (2, 5).

24. For the same function used in the previous problem, find the area under the curve between $x=2$ and $x=5$.

Name: ______________________

AP Physics C Summer Work Answer Sheet

Vector Math (21)

Pythagorean Theorem

1. ________________________
2. ________________________
3. A B C D E
4. ________________________
5. ________________________
6. ________________________

Trig Functions & Right Triangles

7. ________________________
8. ________________________
9. ________________________
10. ________________________

Vector Addition

11. ________________________
12. A B C D
13. A B C D E
14. ________________________
15. ________________________

Components of Vectors

16. ________________________
17. ________________________
18. ________________________
19. ________________________
20. ________________________
21. ________________________

Proportional Reasoning (13)

Proportional Reasoning

1. ________________________
2. ________________________
3. ________________________
4. ________________________
5. A B C D E
6. A B C D E

Parent Graphs

7. A B C D E F
8. A B C D E F
9. A B C D E F G H
10. A B C D E F G H
11. A B C D E F G H
12. A B C D E F G H
13. 1. _______ 2. ________ 3.__________ 4. __________

Vector Math

21 questions

Pythagorean Theorem

Learning Goal:
To understand and apply the Pythagorean Theorem.

The Pythagorean Theorem is named after a religious school from ancient Greece whose students believed whole numbers to be the foundations of the universe. They discovered much interesting math using whole numbers. However, the discovery that they are most famous for also led to the downfall of their religion! The Pythagorean Theorem leads directly to the discovery of irrational numbers—numbers that cannot be written as the ratio of two whole numbers. Seeing that even something as simple as the diagonal of a square leads to irrational numbers shattered their belief in the holiness of whole numbers, but this insight also laid the foundation for many of the discoveries that made Greek mathematics, particularly geometry, so successful.

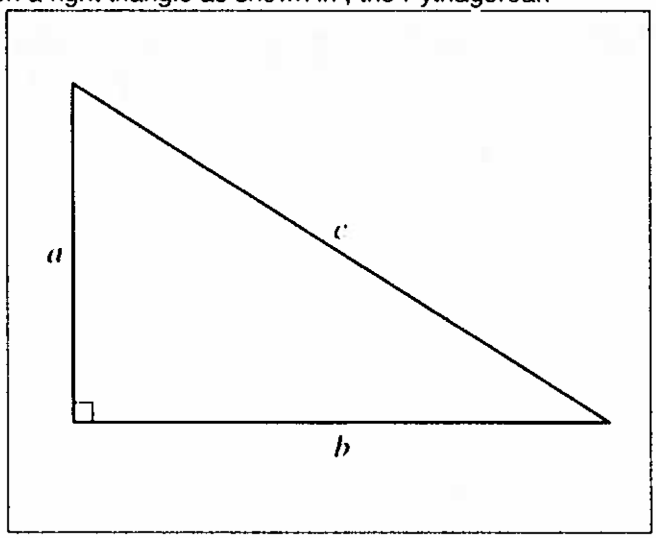
The Pythagorean Theorem relates the lengths of the two legs (the sides opposite the two acute angles) a and b of a right triangle to the length of the hypotenuse (the side opposite the right angle) c. Given a right triangle as shown in , the Pythagorean Theorem is written

$$a^2 + b^2 = c^2.$$

For instance, if you had a right triangle with legs both of length 1 (i.e., $a = b = 1$), then the Pythagorean Theorem would give

$$c^2 = 1^2 + 1^2 = 2,$$

so that $c = \sqrt{2}$.



Part A

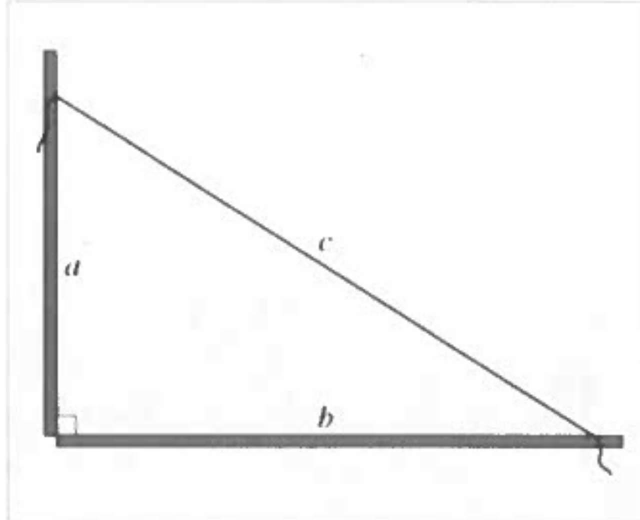
Now, consider a right triangle with legs of lengths 5 cm and 12 cm. What is the length c of the hypotenuse of this triangle?

Express your answer to three significant figures.

ANSWER:

1. c = cm

One application of the Pythagorean Theorem is in making sure that two boards are perpendicular (form a 90° angle) when doing construction, carpentry, etc. If you have two boards and measure out distances on each (marked in blue on), then you can adjust the angle between them until the length of the string is equal to the hypotenuse predicted by the Pythagorean Theorem. At that point, the angle between the boards must be a right angle, to the level of precision that you've measured the lengths.



Part B

Suppose that you have measured a length of 6 cm on one board and 8 cm on the other. You would adjust the two boards until the length of the string had value c to ensure that the boards made a right angle. What is c?

Express your answer in centimeters to three significant figures.

ANSWER:

2. c = cm

Part C

Use the Pythagorean Theorem to determine which of the following give the measures of the legs and hypotenuse of a right triangle.

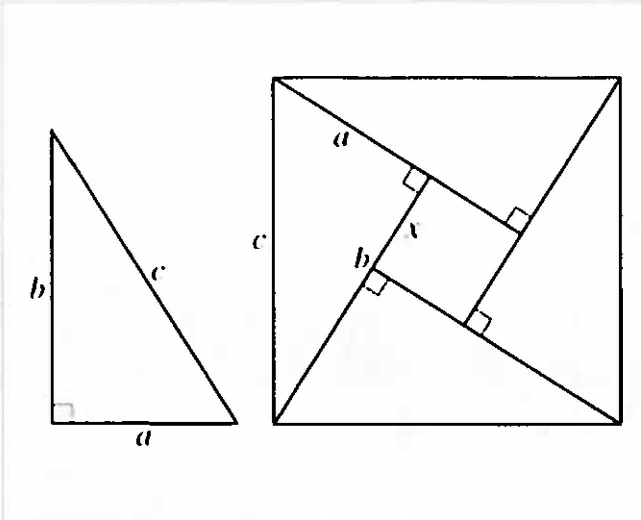
Check all that apply.

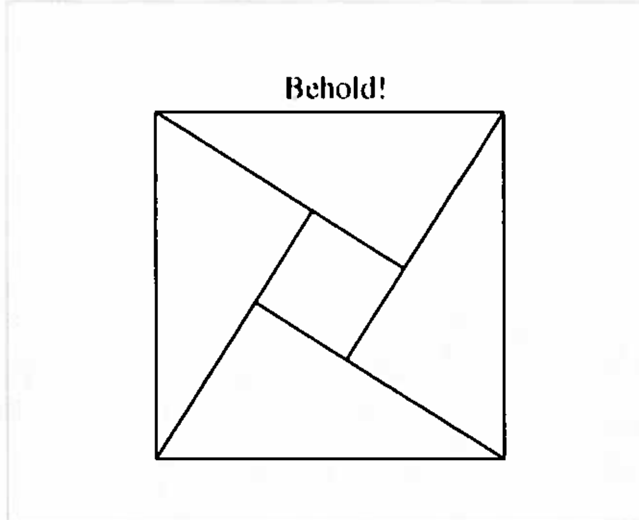
ANSWER:

3.
- A 3, 4, 5
- B 4, 11, 14
- C 9, 14, 17
- D 8, 14, 16
- E 8, 15, 17

Being one of the most famous theorems in all of geometry, many different proofs of the Pythagorean Theorem have been found. One of the most interesting was given by the Indian mathematician Bhaskara, who lived between 1114 and 1185. The entire proof consisted of a single cryptic image as shown in .

To try to understand this proof, let us consider the specific details in Bhaskara's drawing. makes it clear that the original square is made up of four identical right triangles and a smaller square.





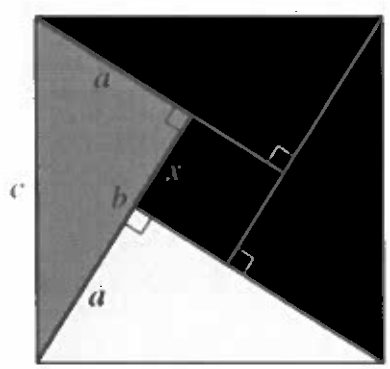
Part D

What is the length x of a side of the small inner square?

Express your answer in terms of the variables a and b.

Hint 1. A more helpful figure

In the figure below, the side of length a for the yellow triangle plus the side of the small square makes up the side of length b for the light blue triangle. You can set up a simple equation relating a, b, and x using this fact. Solve this equation for x.



ANSWER:

4. x =

Part E

Given that the side of the square has a length $b - a$, find the area of one of the four triangles and the area of the small inner square.

Give the area of one of the triangles followed by the area of the small inner square separated by a comma. Express your answers in terms of the variables a and b.

ANSWER:

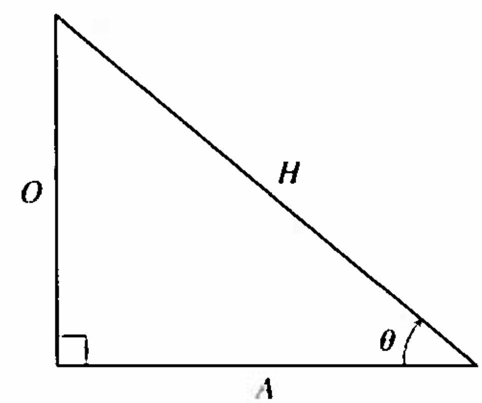
5. $A_{\text{triangle}}, A_{\text{square}}$ =

Trig Functions and Right Triangles

Learning Goal:
To use trigonometric functions to find sides and angles of right triangles.

The functions sine, cosine, and tangent are called *trigonometric* functions (often shortened to "trig functions"). Trigonometric just means "measuring triangles." These functions are called trigonometric because they are used to find the lengths of sides or the measures of angles for right triangles. They can be used, with some effort, to find measures of any triangle, but in this problem we will focus on right triangles. Right triangles are by far the most commonly used triangles in physics, and they are particularly easy to measure.

The sine, cosine, and tangent functions of an acute angle in a right triangle are defined using the relative labels "opposite side" O and "adjacent side" A. The hypotenuse H is the side opposite the right angle.



As you can see from the figure, the opposite side O is the side of the triangle not involved in making the angle. The side called the adjacent side A is the side involved in making the angle that is *not* the hypotenuse. (The hypotenuse will always be one of the two sides making up the angle, because you will always look at the acute angles, not the right angle.)

The sine function of an angle θ, written $\sin(\theta)$, is defined as the ratio of the length O of opposite side to the length H of the hypotenuse:

$$\sin(\theta) = \frac{O}{H}.$$

You can use your calculator to find the value of sine for any angle. You can then use the sine to find the length of the hypotenuse from the length of the opposite side, or vice versa, by using the fact that the previous formula may be rewritten in either of the following two forms:

$$O = H\sin(\theta)$$

or

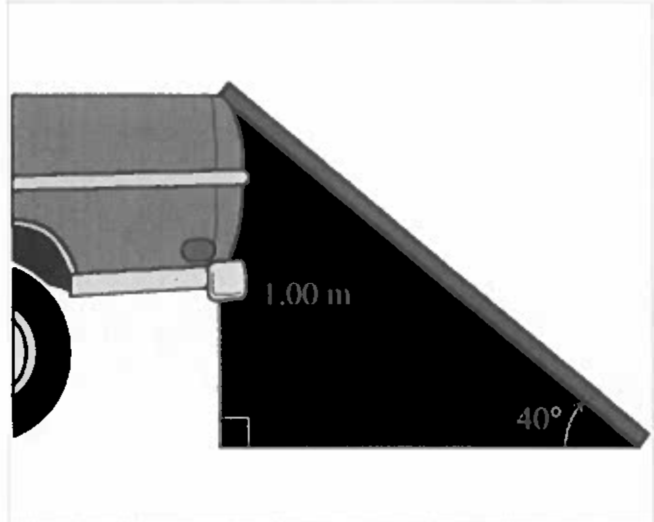
$$H = \frac{O}{\sin(\theta)}.$$

Part A

Suppose that you need to get a heavy couch into the bed of a pickup truck. You know the bed of the truck is at a height of 1.00 m and you need a ramp that makes an angle of 40° with the ground if you are to be able to push the couch.

Use the sine function to determine how long of a board you need to use to make a ramp that just reaches the 1.00-m high truck bed at a 40° angle to the ground.

Express your answer in meters to three significant figures.



ANSWER:

6. m

The cosine function is another useful trig function. The definition of the cosine function is similar to the definition of the sine function:

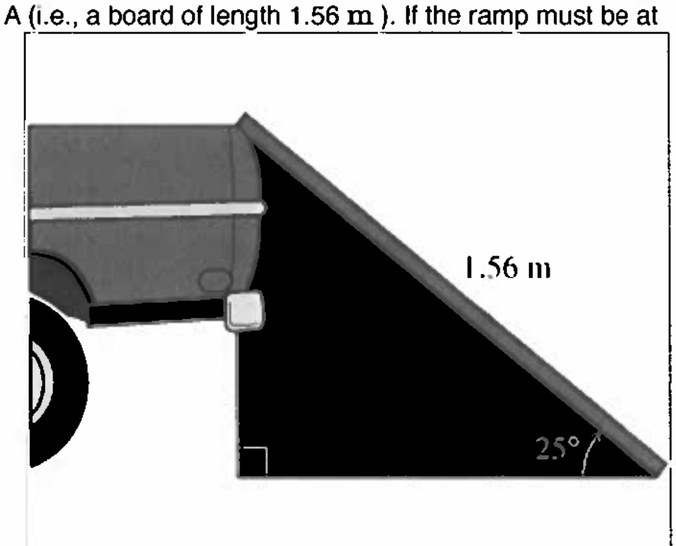
$$\cos(\theta) = \frac{A}{H}.$$

This equation can be rearranged the same way that the equation for sine was rearranged. With the cosine of an angle, you can find the length of the adjacent side from the length of the hypotenuse, or vice versa.

Part B

You need to set up another simple ramp using the board from Part A (i.e., a board of length 1.56 m). If the ramp must be at a 25° angle above the ground, how far back from the bed of the truck should the board touch the ground? Assume this is a different truck than the one from Part A.

Express your answer in meters to three significant figures.



Hint 1. Using the cosine function

The ramp is the *hypotenuse* of the right triangle in the figure, and the distance along the ground is *adjacent* to the 25° angle. To find the length of the adjacent side, use the

$$A = H\cos(\theta)$$

form of the cosine formula. Plugging in the given values will give you the distance along the ground.

ANSWER:

7. m

The third frequently used trig function is the tangent function. The tangent of an angle θ is defined by the equation

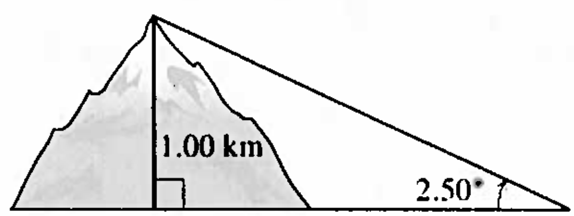
$$\tan(\theta) = \frac{O}{A}.$$

This equation can be rearranged the same way that the equations for sine and cosine were rearranged previously. With the tangent of an angle, you can find the length of the adjacent side from the length of the opposite side or vice versa.

Part C

Surveyors frequently use trig functions. Suppose that you measure the angle from your position to the top of a mountain to be 2.50°. If the mountain is 1.00 km higher in elevation than your position, how far away is the mountain?

Express your answer in kilometers to three significant figures.



Hint 1. Using the tangent function

The height of the mountain is *opposite* the 2.50° angle of the right triangle in the figure, and the distance to the mountain is *adjacent* to the 2.50° angle. To find the distance to the mountain, use the

$$A = \frac{O}{\tan(\theta)}$$

form of the tangent formula. Plugging in the given values will give you the distance to the mountain.

ANSWER:

8. km

All of the trig functions also have inverses. The inverses of the sine, cosine, and tangent functions are written as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, respectively. [Be careful not to confuse the notation $\sin^{-1}(x)$ for the inverse sine function with $(\sin(x))^{-1} = 1/\sin(x)$.] These inverse functions are also sometimes written asin, acos, and atan, short for arcsine, arccosine, and arctangent, respectively. Your calculator should have three buttons with one of those sets of three labels.

Since a trig function takes an angle and gives a ratio of sides, the inverse trig functions must take as input a ratio of sides and then give back an angle. For example, if you know that the length of the side adjacent to a particular angle θ is 12 cm and the length of the hypotenuse of this triangle is 13 cm, you can find the measure of angle θ using the inverse cosine. The cosine of θ would be 12/13, so the inverse cosine of 12/13 will give the value of θ:

$$\cos(\theta) = \frac{12}{13}$$

implies that

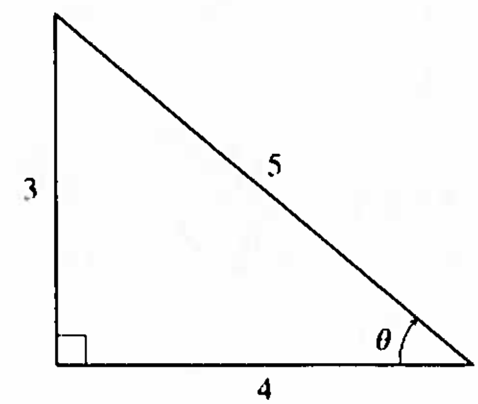
$$\theta = \mathrm{acos}\left(\frac{12}{13}\right).$$

Using the $\cos^{-1}$ or acos button on your calculator, you should check that the measure of θ is 22.6°.

Part D

The 3-4-5 right triangle is a commonly used right triangle. Use the inverse sine function to determine the measure of the angle opposite the side of length 3.

Express your angle in degrees to three significant figures.



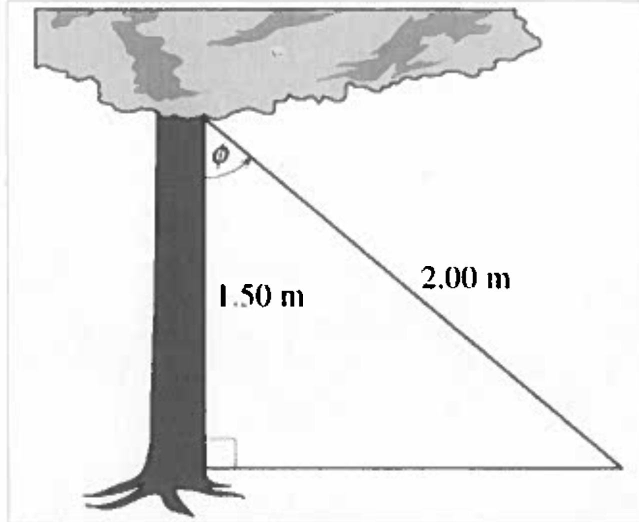
ANSWER:

9. θ = degrees

Part E

A support wire is attached to a recently transplanted tree to be sure that it stays vertical. The wire is attached to the tree at a point 1.50 m from the ground, and the wire is 2.00 m long. What is the angle ϕ between the tree and the support wire?

Express your answer in degrees to three significant figures.



ANSWER:

10. $\phi =$ degrees

Understanding Vector Addition

Learning Goal:
To learn to add vectors.

In physics, many important quantities—from the simple foundations of mechanics such as position and force to very foreign ideas such as electric current densities and magnetic fields—are vectors. Simply knowing what a vector is helps understanding, but to really use the ideas of physics and predict things, you need to be able to do calculations with those vectors. The simplest operation you might need to perform on vectors is to add them.

Suppose that you are swimming in a river while a friend watches from the shore. In calm water, you swim at a speed of 1.25 $\mathrm{m/s}$. The river has a current that runs at a speed of 1.00 $\mathrm{m/s}$.

Note that speed is the magnitude of the velocity vector. The velocity vector tells you both how fast something is moving and in which direction it is moving.

Part A

If you are swimming upstream (i.e., *against* the current), at what speed does your friend on the shore see you moving?

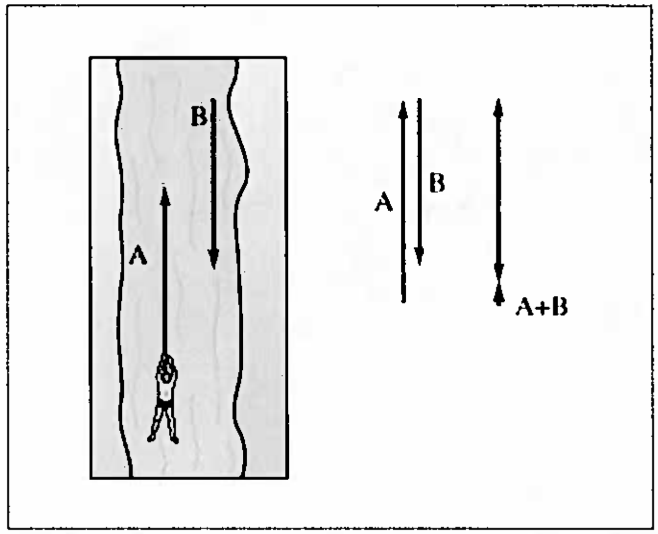
Express your answer in meters per second.

ANSWER:

11. $\mathrm{m/s}$

You likely could answer the last question without thinking about vectors at all. If a person swims against a current, it slows the person down. The speeds subtract in this case, because you are not actually adding speeds. You are adding velocities, which are vectors.

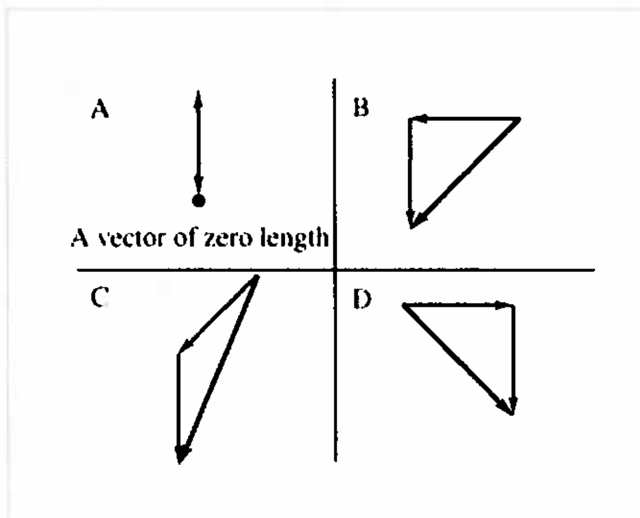
To add two vectors, say $\vec{A} + \vec{B}$, think of taking one vector ($\vec{B}$) and putting its tail on the head of the other vector ($\vec{A}$). The sum of the two vectors is then the vector that begins at the tail of $\vec{A}$ and ends at the head of $\vec{B}$.



Part B

If instead of swimming against the current you swam directly *across* the river (by your reckoning) at a speed of 1.00 $\mathrm{m/s}$ from left to right, which figure correctly shows the velocity vector with which your friend on the shore would see you moving?

Choose the correct figure. The added vectors are shown in gray; the vector representing their sum is shown in black.



ANSWER:

12.

- A
- B
- C
- D

Although drawing vectors is helpful for visualizing what happens when you add vectors, it is not a convenient way to calculate precise results: Adding components is preferable. When you add two vectors, the resulting vector's components are the sums of the corresponding components of the original vectors.

For instance, consider the two vectors $\vec{A}$ and $\vec{B}$ with components (a_x, a_y) and (b_x, b_y), respectively. If you want to find the sum $\vec{A}+\vec{B}$, then you would simply add the *x* components to get the resulting *x* component and add the *y* components to get the resulting *y* component:

$$\vec{A}+\vec{B}=(a_x+b_x, a_y+b_y).$$

For the situation of you swimming *across* the river from left to right at 1.00 $\mathrm{m/s}$, use a standard set of coordinates where the *x* axis is horizontal, with positive pointing to the right, and the *y* axis is vertical, with positive pointing upward.

Part C

Which of the following gives the correct components for the current velocity and the pure swimming velocity (i.e., the velocity that you would have in still water) using this coordinate system?

ANSWER:

A current: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$; swimming: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$

B current: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$; swimming: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

C current: $(-1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$; swimming: $(0.00\ \mathrm{m/s}, -1.00\ \mathrm{m/s})$

D current: $(0.00\ \mathrm{m/s}, 1.00\ \mathrm{m/s})$; swimming: $(-1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

13 E current: $(0.00\ \mathrm{m/s}, -1.00\ \mathrm{m/s})$; swimming: $(1.00\ \mathrm{m/s}, 0.00\ \mathrm{m/s})$

Part D

What is the resultant velocity vector when you add your swimming velocity and the current velocity?

Give the *x* and *y* components in meters per second separated by a comma.

ANSWER:

14. $\mathrm{m/s}$

Part E

Consider the two vectors $\vec{C}$ and $\vec{D}$, defined as follows:

$$\vec{C}=(2.35, -4.27) \text{ and } \vec{D}=(-1.30, -2.21).$$

What is the resultant vector $\vec{R}=\vec{C}+\vec{D}$?

Give the *x* and *y* components of $\vec{R}$ separated by a comma.

ANSWER:

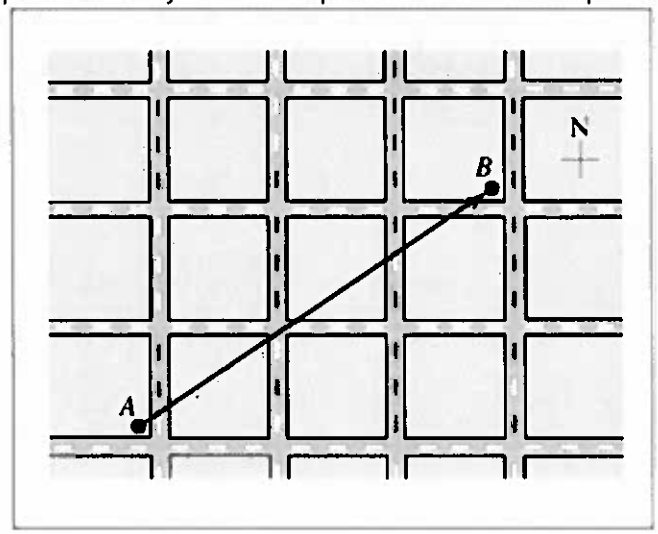
15. R_x, R_y =

Understanding Components of Vectors

Learning Goal:
To understand and be able to calculate the components of vectors.

You have heard vectors defined as quantities with magnitude and direction, familiar ideas also found in statements such as "three miles northeast of here." Components, the lengths in the *x* and *y* directions of the vector, are a different way to define vectors. In this problem, you will learn about components, by considering ways that they arise in everyday life.

Suppose that you needed to tell some friends how to get from point A to point B in a city. The net displacement vector from point A to point B is shown in the figure. You could tell them that to get from A to B they should go 3.606 blocks in a direction 33.69° north of east. However, these instructions would be difficult to follow, considering the buildings in the way.



Part A

You would more likely give your friends a number of blocks to go east and then a number of blocks to go north. What would these two numbers be?

Enter the number of blocks to go east, followed by the number of blocks to go north, separated by a comma.

ANSWER:

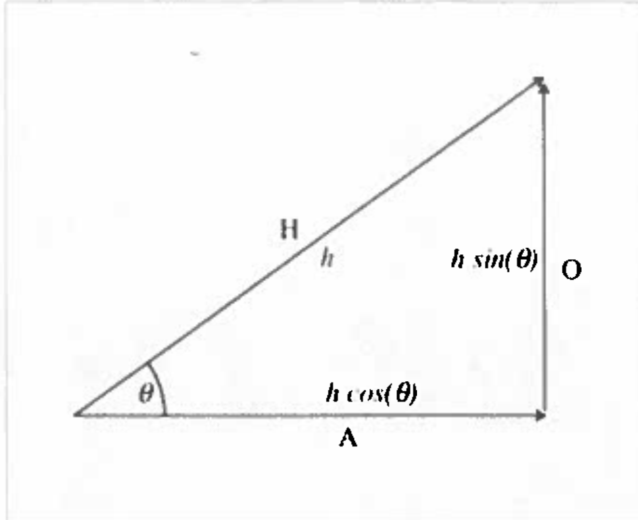
16.

blocks

Notice that the figure with the component vectors drawn in has the shape of a right triangle. You can use trigonometry to find the components of any vector.

Recall that, for some angle θ in a right triangle, the sine of that angle, $\sin(\theta)$, is defined as the length of the side (O) opposite the angle divided by the length of the hypotenuse (H) of the triangle, and the cosine of the angle, $\cos(\theta)$, is defined as the length of the side (A) adjacent to the angle divided by the length of the hypotenuse of the triangle.

In terms of these definitions, and the hypotenuse's length h, the triangle's sides have the following lengths: $h\sin(\theta)$ for the side opposite the angle, and $h\cos(\theta)$ for the side adjacent to the angle.



Part B

Consider the vector $\vec{b}$ with magnitude 4.00 m at an angle 23.5° north of east. What is the x component b_x of this vector?

Express your answer in meters to three significant figures.

ANSWER:

7. b_x = m

Part C

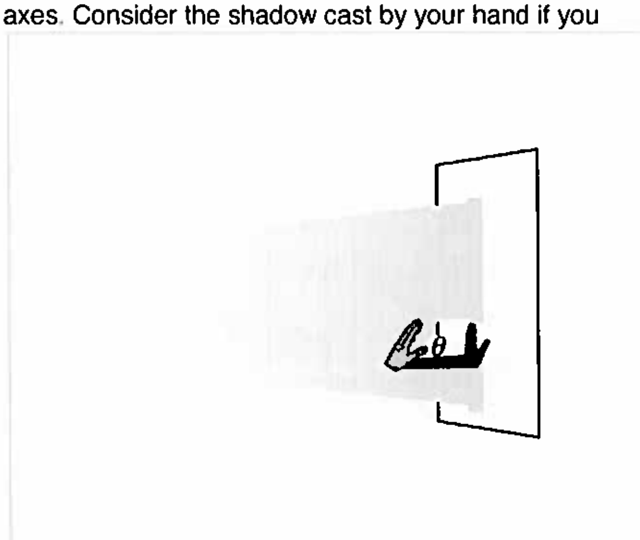
Consider the vector $\vec{b}$ with length 4.00 m at an angle 23.5° north of east. What is the y component b_y of this vector?

Express your answer in meters to three significant figures.

ANSWER:

18. b_y = m

You can also think about components as projections onto the coordinate axes. Consider the shadow cast by your hand if you hold it near a movie screen. As you tilt your hand closer to the horizontal, the shadow gets smaller. If you think of your hand as a vector with tail at the base of your palm and arrow at your fingertips, the shadow's height corresponds to the *y* component of the vector. If you were to shine another light from above, the shadow cast below your hand would correspond to the *x* component.



Part D

What is the length of the shadow cast on the vertical screen by your 10.0 cm hand if it is held at an angle of $\theta = 30.0^\circ$ above horizontal?

Express your answer in centimeters to three significant figures.

Hint 1. Which component do you need?

Since the shadow is vertical and the horizontal direction coincides with the positive *x* axis, the shadow of your hand would be the *y* component of the "hand vector."

ANSWER:

19. | cm |

You can also use your knowledge of right triangles to solve the problem in reverse, that is, to find the magnitude and direction of a vector from its components.

If you know the two components of a two-dimensional vector, you can use the Pythagorean Theorem to find the vector's magnitude (i.e., length) by adding the squares of the two components and then taking the square root. In Parts B and C, the two components were 3.668 m and 1.595 m , and the vector's magnitude is

$$|\vec{v}| = \sqrt{(3.668)^2 + (1.595)^2} = 4.000.$$

Part E

What is the magnitude of a vector with components (15 m, 8 m)?

Express your answer in meters.

Hint 1. More about the Pythagorean Theorem

Recall from geometry that the Pythagorean Theorem says

$$a^2 + b^2 = c^2,$$

where a and b are the lengths of the two legs of a right triangle and c is the length of the hypotenuse. You know from the previous discussion that the *x* and *y* components of a vector can be thought of as the lengths of two legs of a right triangle with the vector itself as hypotenuse. Therefore the magnitude of the vector $|\vec{v}|$ and the two components v_x and v_y must satisfy the Pythagorean Theorem:

$$v_x^2 + v_y^2 = v^2.$$

Taking the square root of both sides gives the relation

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}.$$

ANSWER:

20. | m |

Finding the direction from the components requires a bit of trigonometry. In a right triangle, the tangent of an angle is the length of the side (O) opposite the angle divided by the length of the side (A) adjacent to the angle.

Using this definition and the figure showing the right triangle, you can see that the tangent of the angle above the positive *x* axis is the *y* component of the vector (the length of side O) divided by the *x* component of the vector (the length of side A):

$$\tan(\theta) = \frac{v_y}{v_x}.$$

When you use this formula, remember that you are finding the angle measured counterclockwise from the positive *x* axis. Sometimes you will be asked for the angle with other axes. You should be able to use the same trigonometry described here, but this formula may not be quite right.

Part F

What is the angle above the *x* axis (i.e., "north of east") for a vector with components ($15\ \mathrm{m}$, $8\ \mathrm{m}$)?

Express your answer in degrees to three significant figures.

ANSWER:

21. degrees

13 questions

Proportional Reasoning

Learning Goal:
To understand proportional reasoning for solving and checking problems.

Proportional reasoning involves the ability to understand and compare ratios and to produce equivalent ratios. It is is a very powerful tool in physics and can be used for solving many problems. It's also an excellent way to check answers to most problems you'll encounter. Proportional reasoning is something you may already do instinctively without realizing it.

Part A

You are asked to bake muffins for a breakfast meeting. Just as you are about to start making them, you get a call saying that the number of people coming to the meeting has doubled! Your original recipe called for three eggs.How many eggs do you need to make twice as many muffins?

Express your answer as an integer.

ANSWER:

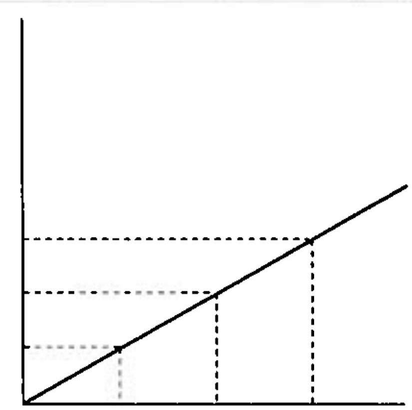
1.

Linear relationships

Although this was a particularly simple example, you used proportional reasoning to solve this problem. It makes sense that if you need twice as many muffins, then you'd need twice as many eggs to make them. We say that the number of eggs is *linearly proportional* to the number of muffins. This sort of relationship is characterized by an equation of the form $y = kx$, where y and x are the two quantities being related (eggs and muffins here) and k is some constant. In a situation where the constant k is not important, we may write $y \propto x$, which means "y is proportional to x".

Writing $(\text{number of muffins}) \propto (\text{number of eggs})$ means we know that if the number of eggs triples, then the number of muffins triples as well. Or, if the number of muffins is divided by 5, then the number of eggs is divided by 5.

The figure shows a graph of $y = kx$ for some constant k. You can see that when you double or triple the original x value, you get double or triple the y value, respectively. Keep this graph in mind and relate it to your intuitive sense as you solve the next problem.



Part B

You have a dozen eggs at home, and you know that with them you can make 100 muffins. If you find that half of the eggs have gone bad and can't be used, how many muffins can you make?

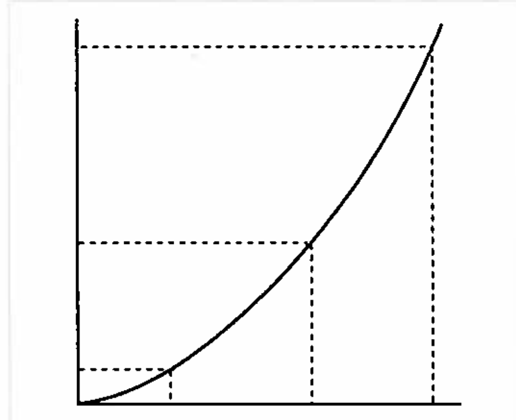
Express your answer as an integer.

ANSWER:

2.

Quadratic relationships

Quadratic relationships are also important in physics and many other areas. In a quadratic relationship, if one number is increased by a factor of a, then the other is increased by a factor of a^2. An example would be the relationship between area and radius of a circle. You know from geometry that $A = \pi r^2$. Since π is a constant, you can rewrite this equation as $A \propto r^2$, which says that A is proportional to *the square of* r. The relation $y \propto x^2$ applies to any equation of the form $y = kx^2$. The figure shows a graph of $y = kx^2$ for some constant k. You can see that when you double or triple the original x value, you get four or nine times the y value, respectively.



Part C

When sizes of pizzas are quoted in inches, the number quoted is the diameter of the pizza. A restaurant advertises an 8-inch "personal pizza." If this 8-inch pizza is the right size for one person, how many people can be fed by a large 16-inch pizza?

Express your answer numerically.

Hint 1. How to approach the problem

The area of a pizza is what determines how many people can be fed by the pizza. You know that the area of a circle is proportional to the square of the radius. Since the radius is proportional to the diameter, it follows that the area is also proportional to the square of the diameter: $A \propto d^2$. Use this relation to determine how the area, and therefore the number of people fed, changes.

ANSWER:

3.

The stopping distance is how far you move down the road in a car from the time you apply the brakes until the car stops. Stopping distance D is proportional to the square of the initial speed v at which you are driving: $D \propto v^2$.

Part D

If a car is speeding down a road at 40 $\mathrm{miles/hour\ (mph)}$, how long is the stopping distance D_{40} compared to the stopping distance D_{25} if the driver were going at the posted speed limit of 25 mph?

Express your answer as a multiple of the stopping distance at 25 mph. Note that D_{25} is already written for you, so just enter the number.

Hint 1. Setting up the ratio

Since $40/25 = 1.6$, the car is moving at a speed 1.6 times the speed limit of 25 mph. The stopping distance is proportional to the square of the initial speed, so the stopping distance will increase by a factor of the square of 1.6.

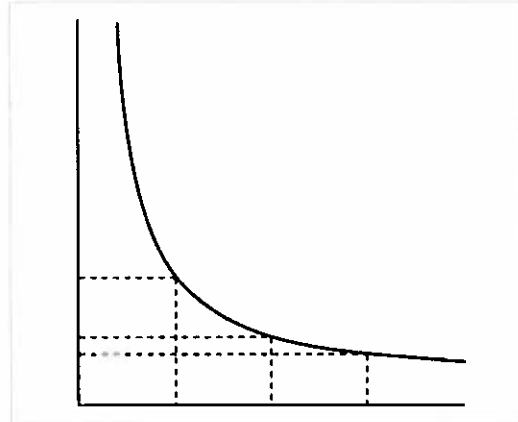
ANSWER:

4. $D_{40} =$ $\times D_{25}$

Inverse relationships

A third important type of proportional relationship is the inverse relationship. In an inverse relationship, as one variable increases the other decreases and vice versa. For instance, if you had a $10 gift certificate to a chocolate shop, then the amount of chocolate that you could get would be inversely proportional to the price of the chocolate you picked. If you buy the $0.25 candies, you could get 40 of them, but if you opt to purchase candies whose price is higher by a factor of 4 ($1.00), then you must reduce the number that you get by a factor of 4 (to 10). Similarly, if the price decreases by a factor of 5 (to $0.05), then you increase the number by a factor of 5 (to 200).
An inverse relationship is based on an equation of the form $y = k/x$, where k is a constant. If y is inversely proportional to x then you would write $y \propto 1/x$ or $y \propto x^{-1}$.

The figure shows a graph of $y = k/x$ for some constant k. You can see that when you double or triple the original x value, you get one-half or one-third times the y value, respectively.



Part E

A construction team gives an estimate of three months to repave a large stretch of a very busy road. The government responds that it's too much inconvenience to have this busy road obstructed for three months, so the job must be completed in one month. How does this deadline change the number of workers needed?

Hint 1. The proportionality

The time to complete the job should be inversely proportional to the number of workers on the job. Therefore, *reducing* the time by a factor of 3 requires *increasing* the number of workers by a factor of 3.

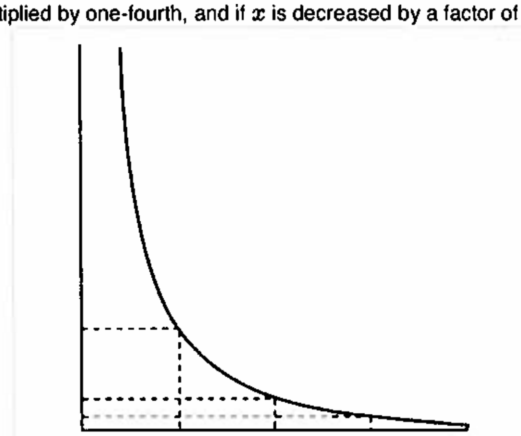
ANSWER:

5.

- A One-ninth as many workers are needed.
- B One-third as many workers are needed.
- C The same number of workers are needed.
- D Three times as many workers are needed.
- E Nine times as many workers are needed.

Inverse-square relationships

All of these proportionalities are in some way familiar to you in your everyday life. There is one other important type in physics with which you may not be as familiar: the inverse-square relationship. The inverse-square relationship is based on an equation of the form $y = k/x^2$, where k is a constant. You would write $y \propto 1/x^2$ or $y \propto x^{-2}$, either of which means "y is inversely proportional to the square of x." Although this may look or sound more intimidating than the relations we've looked at previously, it works in essentially the same way. If x is doubled, then y is multiplied by one-fourth, and if x is decreased by a factor of 2, y is multiplied by 4. The figure shows a graph of $y = k/x^2$ for some k. You can see that when x *increases* by a factor of 2 or 3, y *decreases* by a factor of 4 or 9, respectively.



Part F

The loudness of a sound is inversely proportional to the square of your distance from the source of the sound. If your friend is right next to the speakers at a loud concert and you are four times as far away from the speakers, how does the loudness of the music at your position compare to the loudness at your friend's position?

ANSWER:

6.

A The sound is one-sixteenth as loud at your position.

B The sound is one-fourth as loud at your position.

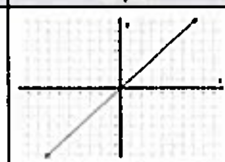
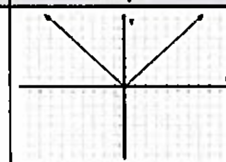
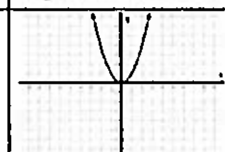
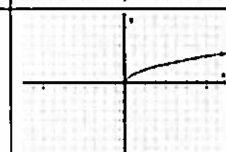
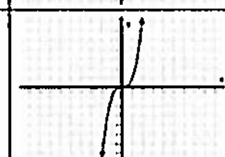
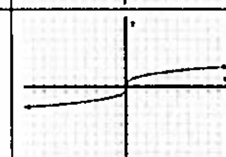
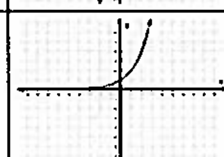
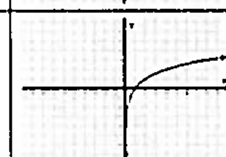
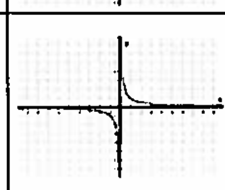
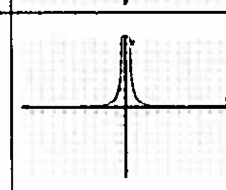
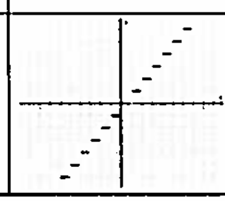
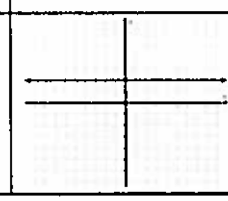
C The sound is equally loud at your position.

D The sound is four times as loud at your position.

E The sound is sixteen times as loud at your position.

Parent Graphs

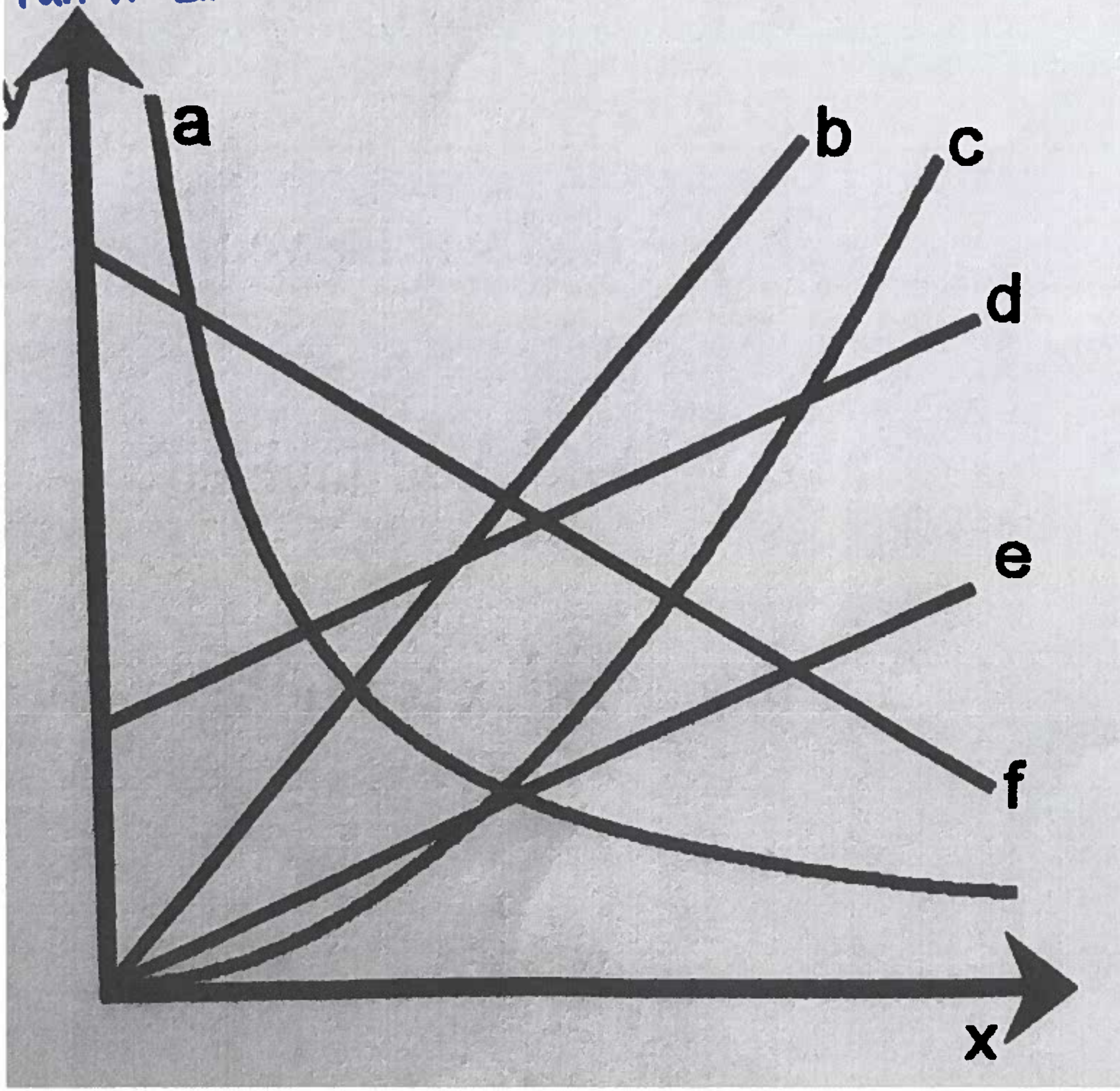
Learning Goal:
Parent graphs.

Parent Function	Graph	Parent Function	Graph
$y=x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\lvert x\rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y=x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y=\sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y=x^3$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$	
$y=b^x,\ b>1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y=\log_b(x),\ b>1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y=\frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y=\frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y=\text{int}(x)=[x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (only integers)		$y=C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$	

Part A

The diagram shows a number of relationships between x and y.

Part A Linear



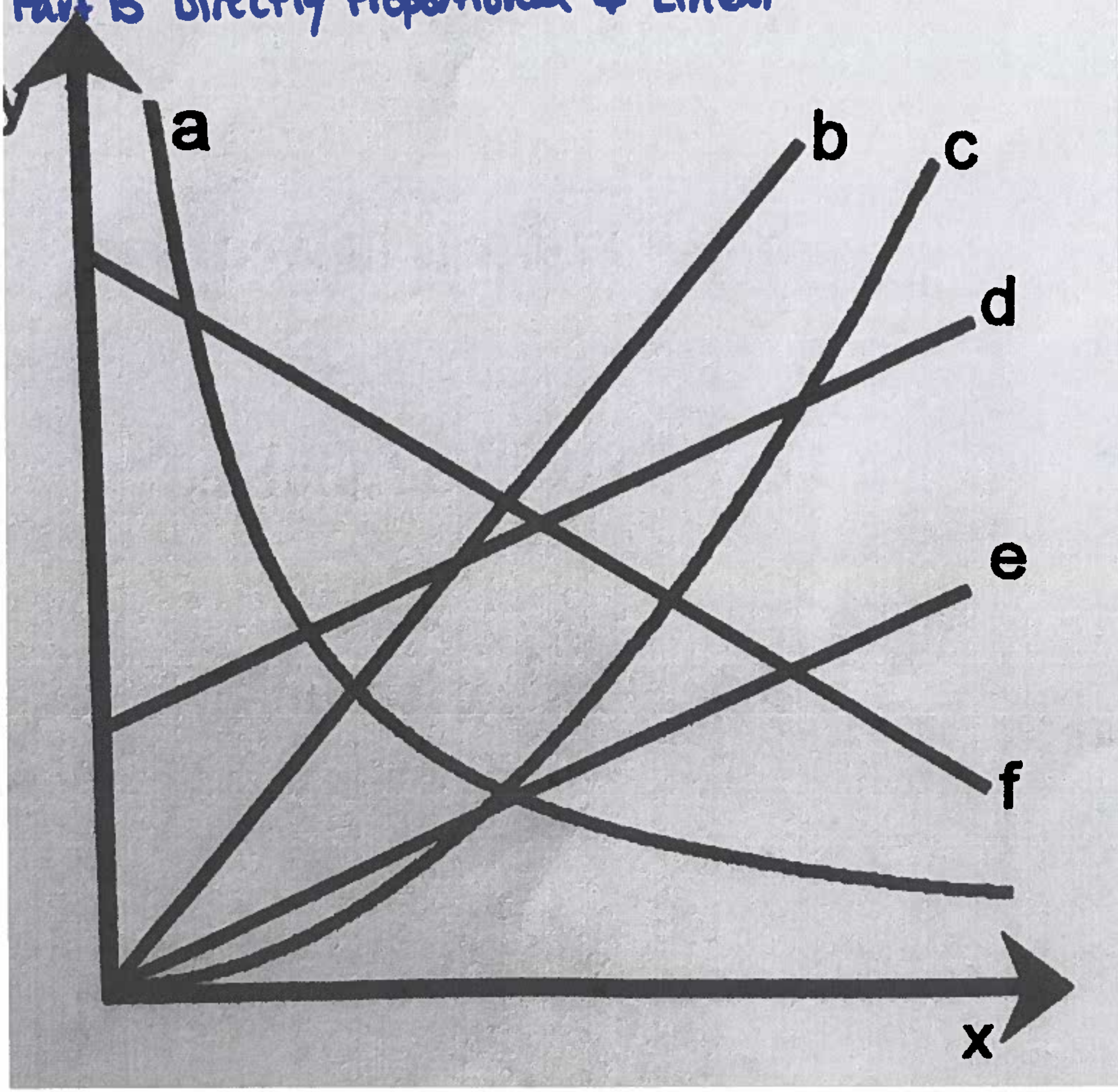
ANSWER:

7. Which relationships are linear?

- [ ] a
- [ ] b
- [ ] c
- [ ] d
- [ ] e
- [ ] f

Part B - Directly Proportional and Linear Relationship

Part B Directly Proportional + Linear



The diagram shows a number of relationships between x and y.

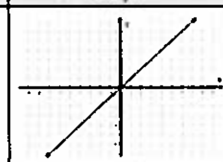
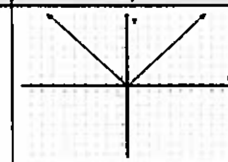
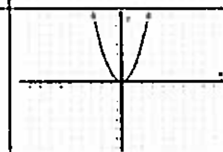
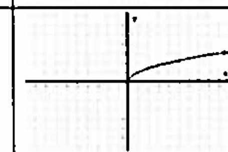
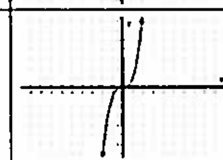
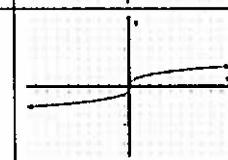
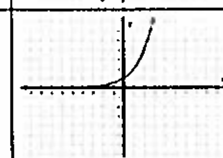
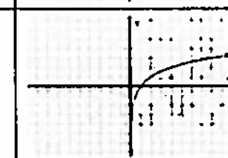
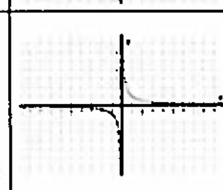
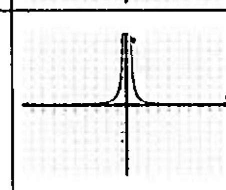
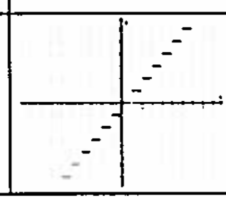
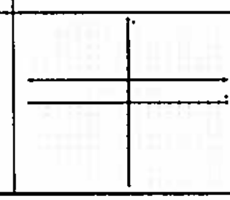
Being directly proportional and having a linear relationship are the same thing when the y-intercept is zero. That means b in y=mx+b is zero.

ANSWER:

8. Which relationships are direct proportions?

- a
- b
- c
- d
- e
- f

Review Parent Graphs

Parent Function	Graph	Parent Function	Graph
$y = x$ Linear Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$		$y = \lvert x \rvert$ Absolute Value Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$	
$y = x^2$ Quadratic Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$		$y = \sqrt{x}$ Square Root Neither Domain: $[0, \infty)$ Range: $[0, \infty)$	
$y = x^3$ Cubic Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$		$y = \sqrt[3]{x}$ Cube Root Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$	
$y = b^x$, $b > 1$ Exponential Neither Domain: $(-\infty, \infty)$ Range: $(0, \infty)$		$y = \log_b(x)$, $b > 1$ Log Neither Domain: $(0, \infty)$ Range: $(-\infty, \infty)$	
$y = \frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$		$y = \frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$	
$y = \text{int}(x) = [x]$ Greatest Integer Neither Domain: $(-\infty, \infty)$ Range: $\{y : y \in \mathbb{Z}\}$ (only integers)		$y = C$ Constant Function Even Domain: $(-\infty, \infty)$ Range: $\{y : y = C\}$	

Part C - Proportional Equations

Choose the following relationships that are direct proportions.

You did not open hints for this part.

ANSWER:

9.

A $y=1/x$
B $y=a/x$
C $y=ax$
D $y=ax^2$
E $y=x$
F $y=ax+b$
G $y=3x$
H $y=a/x^2$

Part D - Proportional Equations

Choose the following relationships that are linear relationships.

You did not open hints for this part.

ANSWER:

10.

A $y=ax^2$
B $y=ax$
C $y=3x$
D $y=ax+b$
E $y=1/x$
F $y=a/x$
G $y=a/x^2$
H $y=x$

Part E - Proportional Equations

Choose the following relationships that are quadratic relationships.

ANSWER:

11.

A $y=a/x$
B $y=a/x^2$
C $y=1/x$
D $y=ax^2$
E $y=ax$
F $y=3x$
G $y=x$
H $y=ax+b$

Part F - Proportional Equations

Choose the following relationships that are inverse square.

ANSWER:

12.

A $y=a/x$
B $y=ax+b$
C $y=a/x^2$
D $y=1/x$
E $y=ax^2$
F $y=ax$
G $y=x$
H $y=3x$

Part G - Mathematical Relationship

For each of the following mathematical relations, state what happens to the value of y when the following changes are made. (k is constant)

ANSWER:

13.

Reset Help

A **quadrupled (x4)**
B **quartered (1/4)**
C **x9**
D **tripled (x3)**
E **halved (1/2)**
F **doubled (x2)**

1. $y = kx$, x is tripled. y is ___ .

2. $y = k/x^2$, x is doubled. y is ___ .

3. $y = kx^2$, x is tripled. y is ___ .

4. $y = k/x$, x is halved. y is ___ .